MATH 425A
SOME COMMON FUNCTIONS

FALL 2019

In this document, I will define some common functions which often come up in math classes.

Power Function: The power function is a common function. It is defined as follows. Let $n \in \mathbb{N}$. For any $x \in \mathbb{R}$, we will write

$$x^n = \underbrace{x \cdot x \cdots x}_{\text{n times}}$$

The above expression is called x to the n^{th} power, or just x to the n. When convenient, we recognize this operation as a function $f : \mathbb{R} \to \mathbb{R}$ with

$$f(x) = x^n$$

A Special Case: It is also frequently convenient to define the following. For any $x \in \mathbb{R}$, we will write

$$x^0 = 1$$

In words, this is the statement that any number x raised to the 0^{th} power is 1. In particular, this means that we have defined $0^0 = 1$.

The Absolute Value: The absolute value of any $x \in \mathbb{R}$ is written:

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

One can check that $|x| \geq 0$ for all $x \in \mathbb{R}$. In fact, $|x| = 0$ if and only if $x = 0$. In words, $|x|$ measures the distance from x to 0 on the real line. Similarly, for any $x, y \in \mathbb{R}$, $|x - y|$ measures the distance from x to y.

Factorials: For any $n \in \mathbb{N}$, the number n factorial, which we denote by $n!$, is defined by

$$n! = n(n-1)\cdots(3)(2)(1)$$

By convention, we set $0! = 1$.

n choose k: For any $n \in \mathbb{N}$ and each $0 \leq k \leq n$, we define n choose k, also called the binomial coefficient, by setting

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$